

# Experimental Investigation of Solid Particles Flow in a Conical Spouted Bed Using Radioactive Particle Tracking

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*Solid particles flow in a conical spouted bed is characterized by radioactive particle tracking. The influence of operating conditions on key parameters of this flow is evaluated and discussed: the morphology of the solid bed is not strongly influenced by the forces exerted by the gas on the solid particles, but rather by geometrical considerations; the particles spend approximately 8% of their time in the spout in all experiments; it is the force exerted on the solid particles by the gas that directly controls the volumetric flow rate between adjacent regions, and not the amount of particles in the bed; as  $U/U_{ms}$  increases, the volume of solid particles in the annulus decreases, the volume of solid particles in the fountain increases and the volume of solid particles in the spout remains constant. Correlations to predict key flow parameters as functions of operating conditions are also established and discussed. © 2015 American Institute of Chemical Engineers AICHE J, 62: 26–37, 2016*

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## Introduction

Spouted beds are gas-solid contactors in which a cyclic solid flow occurs, defining three regions: the spout, the fountain, and the annulus (see Figure 1). They are particularly interesting gas-solid contactors for many applications. For instance, regarding the drying of solid particles, they offer an intrinsic intermittent regime, as the solid particles successively and continuously pass from one region to the other.

In a spouted bed, particles travel rapidly up the spout until they reach a position above the bed surface where they disengage from the gas in the fountain region and then, they move downward in the annulus. A small fraction of the ascending gas passes in the annulus where it percolates through the downwards-moving particles. To initiate this cycling particle movement, the injection gas velocity must be at or above the minimum spouting velocity ( $U_{ms}$ ). Above this specific velocity, the pressure drop through the spouted bed remains for increasing gas inlet velocity.<sup>1</sup>

The conical spouted bed, compared with the more common conico-cylindrical spouted bed, has been reported to allow better operating stability over a wider range of gas flow.<sup>2</sup> Some

characteristic parameters of a conical spouted bed are presented in Figure 1.

There are numerous correlations expressing the minimum spouting velocity of a conical spouted bed ( $U_{ms}$ ).<sup>3</sup> The most utilized correlation is that proposed by Olazar et al.,<sup>1</sup> based on a Reynolds number for  $U_{ms}$  based on the gas inlet diameter ( $D_i$ )

$$Re_{msi} = \frac{U_{ms} D_i \rho}{\mu} = 0.126 Ar^{0.5} \left( \frac{D_b}{D_i} \right)^{1.68} \left[ \tan \left( \frac{\alpha}{2} \right) \right]^{-0.57} \quad (1)$$

where (see Figure 1)  $D_i$  is the gas inlet diameter,  $\rho$  is the gas density,  $\mu$  is the gas dynamic viscosity,  $D_b$  is the diameter of the stagnant bed surface (proportional to  $H_0$ ),  $\alpha$  is the cone angle.  $Ar$  is the Archimedes number defined as follows (with  $\rho \ll \rho_s$ )

$$Ar = \frac{g d_p^3 \rho \rho_s}{\mu^2} \quad (2)$$

where  $g$  is the acceleration of gravity,  $d_p$  is the average particle diameter, and  $\rho$  is the solid particle density.

The research unit at the Universidad del Pais Vasco (Bilbao, Spain) is very active in the field of spouted bed research, with a particular interest in conical spouted bed applications.<sup>4</sup> This group makes use mostly of fiber optical probes for measurements of spouted bed hydrodynamics. They determined operating and geometric conditions to obtain stable flow regimes

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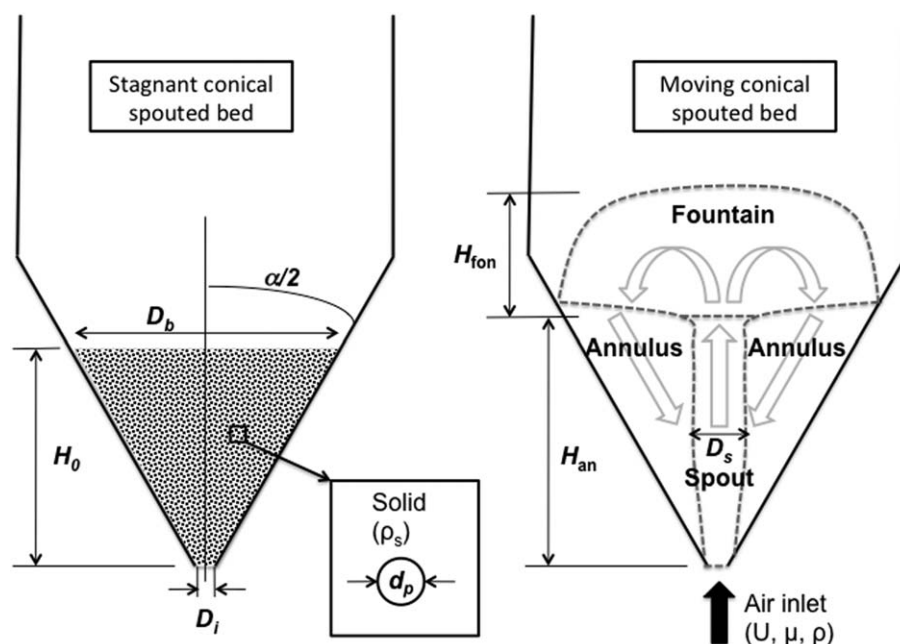


Figure 1. Specific parameters of a static conical bed and an operating conical spouted bed.

in conical spouted beds. They showed that the correlations developed for cylinder-conical spouted beds are not valid for conical spouted beds.<sup>1</sup> They measured particle and gas velocity profiles in conical spouted beds, as well as solid particle circulation rates and patterns.<sup>5,6</sup> They determined how these circulation rates and patterns are influenced by the contactor geometry and operating conditions.<sup>7</sup> They reported correlations expressing the radial and longitudinal profiles of particle velocity in conical spouted beds,<sup>8</sup> together with segregation behavior and pressure drop in the bed.<sup>4</sup> They also studied the transversal flow of solids from the annulus to the spout and they concluded that, in a conical spouted bed, this flow was distributed along the entire spout height.<sup>9</sup>

In several applications, the particle residence time of the solid particles in the different regions of a conical spouted bed may be important information to model the operation of the bed. For instance, regarding drying in a conical spouted bed, it is often assumed that drying only occurs in the spout. Therefore, time spent by the particles in the spout region is a key parameter for modeling of conical spouted bed dryers. Nevertheless, to our knowledge, there is no reported study on the particle residence time in the different regions of a conical spouted bed. Reported experimental results are mostly obtained using intrusive optical fiber probes or static pressure probes, which were shown to generate local perturbations of the gas-solid flow.<sup>10</sup> One improvement would be to use nonintrusive techniques such as tomography<sup>11</sup> or radioactive particle tracking (RPT), the latter of which has been used to study solid particle flow in conico-cylindrical spouted beds<sup>12–14</sup> and conical fluidized beds.<sup>15</sup>

The general objective of this work is to contribute to a better understanding and modeling of the solid particles flow in conical spouted beds. For this purpose, an instrumented laboratory conical spouted bed, with glass beads as solid particles, is used. The solid particles flow taking place in this spouted bed is characterized by means of nonintrusive RPT, which enables

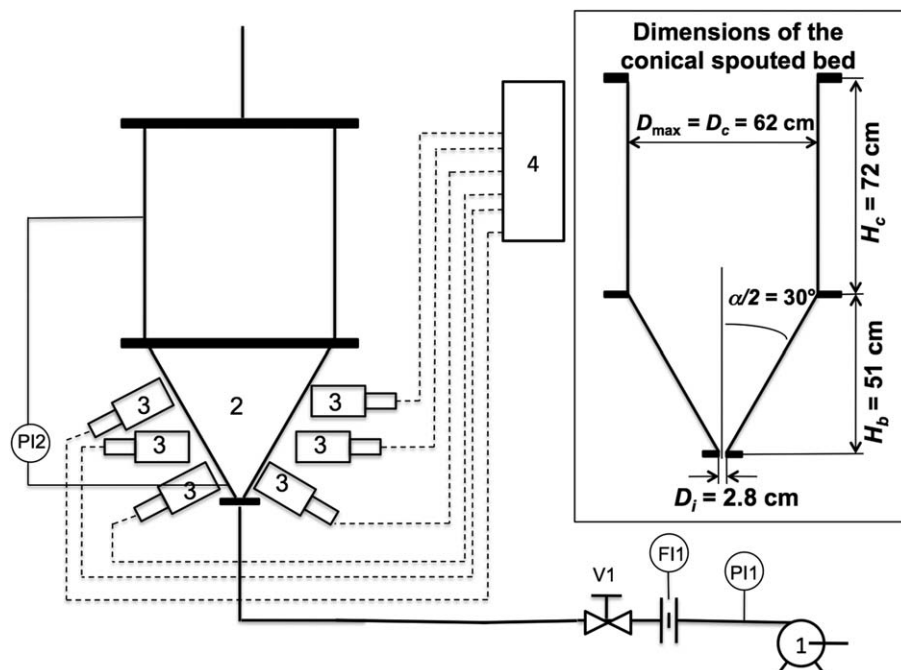
tracking a radioactive tracer that moves along with the glass beads. An original method is developed to derive mean values for key solid particles flow parameters from the time evolution of the tracer position. The results are analyzed to highlight and characterize the key phenomena governing the solid particles flow in the conical spouted bed. Dimensional analysis is used to build correlations predicting the mean residence times of the solid particles in the different regions of the conical spouted bed as functions of operating conditions. These correlations are briefly discussed.

## Materials and Methods

### Experimental setup: Conical spouted bed

The experimental setup is presented schematically in Figure 2. It is composed of a conical spouted bed wherein dry gas is blown. The vessel is composed of a lower stainless steel conical section where the spouting takes place (inlet diameter  $D_i$  of 2.8 cm, cone angle  $\alpha$  of 60°, cone height  $H_b$  of 51 cm, maximum diameter  $D_{max}$  of 62 cm), and an upper stainless steel cylindrical part (cylinder diameter  $D_c$  of 62 cm, cylinder height of  $H_c$  of 72 cm). The same cone angle and inlet diameter were used in a previous study of the solids flow in a cylinder-conical spouted bed.<sup>13</sup> Therefore, this choice makes it possible to compare results obtained in both types of spouted bed.

The gas flow is manually regulated using a pressure gauge (0–100 psig) and a valve, and by reading the pressure drop through an orifice plate (orifice of 0.022 m, pipes with a diameter of 0.041 m) on a water U-manometer. A second water U-manometer measures the pressure drop through the solid particles bed. Nine scintillation detectors (NaI crystal) are installed around the conical part of the bed. They are connected with coaxial cables to an acquisition system composed of an amplifier, a high-speed counter for each detector, and a



**Figure 2.** Experimental setup for the study of the solid flow in a conical spouted bed: gas inlet (1), pressure gauge (PI1), valve (V1), orifice plate including a U-manometer (F11), conical spouted bed (2), U-manometer (PI2), detectors (3), and acquisition system (5).

computer recording the number of counts measured by each detector during each sampling time.

#### **Solid material**

The solid particles chosen for the study of the solid flow in the conical spouted bed are 3-mm glass beads with a density of  $2500 \text{ kg/m}^3$ . The same solid particles were used in a previous study of the flow in a cylinder-conical spouted bed.<sup>13</sup> Therefore, this choice allows for a comparison of the results obtained in both types of spouted bed.

#### **Experimental conditions**

The same conical spouted bed is used for all the experiments. The influence of two operating parameters on the solid flow in this conical spouted bed is studied. These two operating parameters are the static bed height ( $H_0$ ) and the relative gas injection velocity, defined as the injection velocity ( $U$ ) divided by the minimum spouting velocity ( $U_{ms}$ ; see Figure 1).  $U$  and  $U_{ms}$  are defined as superficial velocities calculated at the bed inlet ( $D_i$ ) under normal conditions (Nm/s:  $20^\circ\text{C}$  and 1 atm). The considered ranges of operating parameter values are presented in Table 1. The rationale is to consider a similar range of  $U/U_{ms}$  values for the three different values of  $H_0$ . However, due to the limitation of the air compressor used for the experiments, it was not possible to cover the full range of  $U/U_{ms}$  values for the highest value of  $H_0$ . The geometrical configuration (dimensions, angle) of the conical spouted bed, the inlet air properties, and the solid particles properties are kept constant for all experiments.

#### **RPT technique**

The tracer is one sealed 3-mm glass bead filled with a small mass of Scandium ( $^{46}\text{Sc}$ ). Therefore, the tracer has the same size and approximately the same density as the solid particles constituting the bed. Tracking this tracer is then equivalent to

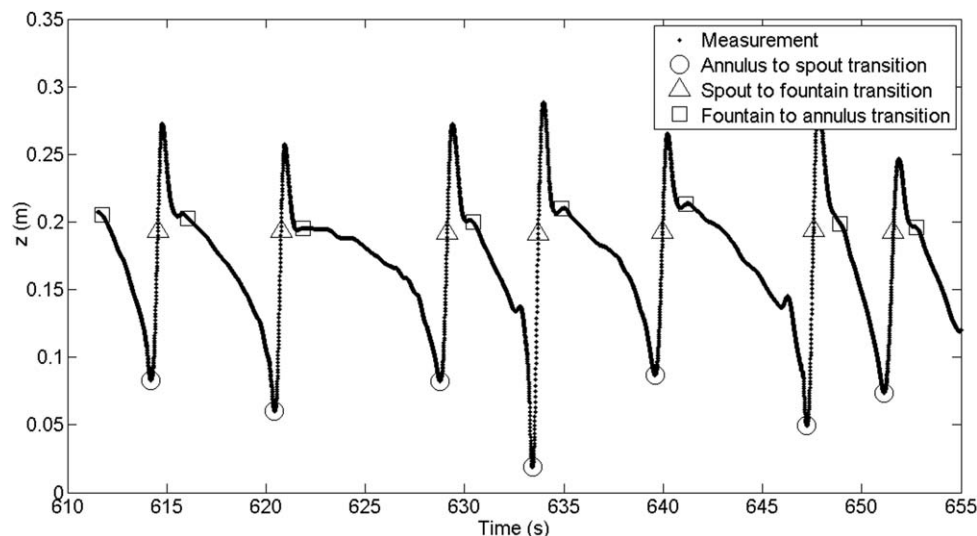
tracking one of these solid particles. Scandium ( $^{46}\text{Sc}$ ) is activated during approximately 3 h by irradiation of  $^{46}\text{Sc}$  in the Slowpoke nuclear reactor at Polytechnique Montréal (Montréal, Canada) to reach a radioactive activity of around 2.6 MBq. As the half-life of  $^{46}\text{Sc}$  is 83.8 days, the radioactive activity of the tracer may thus be considered as constant during an experiment of 1 h.

Nine detectors are used. They are placed around the conical part of the bed and are organized to cover homogeneously the cone region containing the solid particles. They are oriented horizontally or with an inclination such that their faces are parallel to the bed walls (see Figure 2). The normal vectors of their faces always point toward the symmetry axis of the cone. The position and the orientation of these detectors are accurately measured ( $\pm 1 \text{ mm}$ ) before each experiment.

At time  $t$  during an experiment, the tracer position is determined from the counts recorded by each detector during the time interval  $[t, t + 5 \text{ ms}]$ . It allows a good resolution of the tracer position, both in the spout and the fountain. It allows, during a cycle, acquiring the coordinates of more than 50 successive positions of the tracer while it is in the spout (where its velocity is the largest). One single experiment lasts for 1 h. During an experiment, the tracer undergoes more than 500 cycles (spout-fountain-annulus). During preliminary experiments, it was validated that this duration allows determining average values of the solid particles flow parameters in the bed that are independent of the experiment's duration.

**Table 1.** Range of Operating Parameters Used During the Experiments

$H_0$ (m)	$U$ (Nm/s)	$U/U_{ms}$
0.15	40.6–50.8	1.05–1.31
0.20	57.1–71.4	1.04–1.30
0.26	91.9–99.5	1.04–1.12



**Figure 3. Time evolution of the  $z$  coordinate of the tracer position during a typical experiment.**  
Detection of the transition times when the tracer enters each region ( $H_0 = 20$  cm and  $U/U_{ms} = 1.18$ ).

Details of the algorithm used for the determination of the tracer position, from the counts recorded by each detector during 5 ms, have already been published.<sup>16</sup>

As the gamma ray emission has a statistical nature, the reconstructed data exhibit a noise, so that a Savitzky–Golay filter is applied to each coordinate of the tracer position in order to smooth out their time evolution.

Finally, as the vessel has a cylindrical symmetry, the tracer position is expressed as its vertical ( $z$ ) and radial ( $r$ ) coordinates.

### Data treatment

The RPT measurements allow determining the  $r$  and  $z$  coordinates of the tracer position at different sampling times.

The error on the time measurement may be assumed to be less than the sampling time, that is, less than 5 ms. As this value is far smaller than the average times reported later in this article (0.1 – 10 s), the error bars will not be represented for the time values. The accuracy and the precision of the measurement of the tracer position using a similar RPT setup have already been published to be around 2 mm.<sup>17</sup> Again, this value is small compared with the spatial data presented hereafter and it is relatively constant for all measurements; therefore, the error bars for the spatial data will not be represented neither. Moreover, this study focuses on time and space averaged information collected from large amounts of cycles (spout–fountain–annulus); thus, the results presented hereafter are not significantly influenced by the measurement errors and may be considered as valid.

The time evolution of the  $z$  coordinate of the tracer position clearly shows the cyclic behavior of the solid particles flow in the conical spouted bed (Figure 3). During the  $i$ th cycle of an experiment, the transition times of the tracer from one region of the bed to the next one are defined as follows (see Figure 3 for an example on how these transition times are detected):

- Transition time from the annulus to the spout ( $\tau_{sp,i}$ ): when the  $z$  coordinate of the tracer reaches its minimum value within one cycle;
- Transition time from the spout to the fountain ( $\tau_{fo,i}$ ): when, after  $\tau_{sp,i}$ , the  $z$  coordinate of the tracer position becomes

equal to the mean height of the annulus (mean value of the  $z$  coordinate of the tracer when it enters the annulus);

- Transition time from the fountain to the annulus ( $\tau_{an,i}$ ): when, after  $\tau_{fo,i}$ , the second-order derivative of the  $z$  coordinate of the tracer position with respect to time becomes equal to zero for the third time. This definition is motivated by the fact that, after  $\tau_{fo,i}$ , there are three transitions in the vertical movement of the tracer: the first one corresponds to the transition from the upward to the downward movement of the tracer at the top of the fountain; the second one corresponds to the rebound or slip of the tracer on the wall of the conical section or on the bed surface (some rebounds may be observed in Figure 3); and the last transition corresponds to the penetration of the tracer in the bulk of particles in the annulus.

The knowledge of these three transition times for the different cycles of an experiment allows calculating the following key solid particles flow parameters in the conical spouted bed:

- Mean spout radius ( $r_{sp}$ );
- Mean annulus height ( $h_{an}$ );
- Vertical distribution of the mean radius of penetration of the solid particles in the spout: this distribution presents the average value of the radial coordinate of the solid particles at the transition from the annulus to the spout, as a function of the vertical coordinate of the solid particles at this transition;
- Vertical distribution of the cumulative probability of the penetration of the solid particles in the spout: this distribution presents, for every vertical coordinate in the bed, the probability that the solid particles transit from the annulus to the spout at a height inferior or equal to this vertical coordinate;
- Radial distribution of the mean height of penetration of the solid particles in the annulus: this distribution presents the average value of the vertical coordinate of the solid particles at the transition from the fountain to the annulus, as a function of the radial coordinate of the solid particles at this transition;
- Radial distribution of the cumulative probability of the penetration of the solid particles in the annulus: this distribution presents, for every radial coordinate at the bed surface, the probability that the solid particles transit from the fountain to the annulus at a distance from the cone axis inferior or equal to this radial coordinate;



- Mean value of the solid particles residence time in the spout;
- Mean value of the solid particles residence time in the annulus;
- Mean value of the solid particles residence time in the fountain;
- Mean value of the total solid particles cycling time in the bed;
- Global solid particles flow rate circulating in the bed;
- Mean value of the solid particles velocity in the annulus.

These calculations are presented in the following sections. Using these results, the influence of the operating conditions on the key solid particles flow parameters are discussed and quantified.

### Solid bed dimensions

The mean spout radius ( $r_{sp}$ ) is defined as the mean value of the  $r$  coordinate of the tracer position when the tracer enters the spout.

The mean annulus height ( $h_{an}$ ) is defined as the mean value of the  $z$  coordinate of the tracer position when the tracer enters the annulus.

The mean distance traveled by the solid particle in the spout ( $d_{sp}$ ) is defined as the distance between the mean positions of the tracer when it enters the spout and when it enters the fountain. The mean distance traveled by the solid particle in the annulus ( $d_{an}$ ) is defined as the distance between the mean positions of the tracer when it enters the annulus and when it enters the spout.

### Spatial distributions

For a given experiment, the spatial distribution of the penetration of the solid particles in the spout is characterized from the transition times  $\tau_{sp,i}$  and the corresponding tracer positions.

A domain in the  $z$ - $r$  plane, larger than the region where the solid flow takes place, is considered and discretized into  $n \times m$  square elements of side  $\Delta = 5$  mm. This choice of  $\Delta$  allows a good resolution of the calculated distribution. Element  $(j, k)$  of this discretization is centered at  $z = \Delta(j-1/2)$  and  $r = \Delta(k-1/2)$ .

A vector  $\mathbf{R}$  which entries  $R_k$  is then introduced

$$R_k = \Delta \left( k - \frac{1}{2} \right), \quad \forall k, \quad 1 \leq k \leq m \quad (3)$$

$R_k$  is, therefore, the  $r$  coordinate of the center of the  $(j, k)$  element of the discretization of the domain ( $\forall j, 1 \leq j \leq n$ ).

A  $(n, m)$  matrix  $\mathbf{S}$  is also introduced and initialized with  $S_{j,k} = 0$  ( $\forall j, 1 \leq j \leq n$  and  $\forall k, 1 \leq k \leq m$ ). For each transition time  $\tau_{sp,i}$  ( $1 \leq i \leq n_t$ , where  $n_t$  is the number of cycles experienced by the tracer during the experiment),  $S_{j,k}$  is incremented by one if, at time  $\tau_{sp,i}$ , the tracer is in element  $(j, k)$  of the discretization of the domain, that is, if the  $z$  coordinate of the tracer position is in the interval  $[j\Delta, j\Delta]$  and if the  $r$  coordinate of the tracer position is in the interval  $[k\Delta, k\Delta]$ . Therefore,  $S_{j,k}$  represents the amount of tracer transitions from the annulus to the spout, which occurred, during an experiment, at a position included in element  $(j, k)$  of the discretized domain.

If  $\mathbf{P}$  is a column vector of  $n$  entries defined as follows

$$P_j = \frac{1}{\sum_{k=1}^m S_{j,k}} \sum_{k=1}^m S_{j,k} R_k \quad (4)$$

$P_j$  is equal to the mean  $r$  coordinate of the tracer position when the tracer enters the spout with a  $z$  coordinate in the interval  $[j\Delta, j\Delta]$ .

If  $\mathbf{C}$  is a column vector of  $n$  entries defined as follows

$$C_j = \frac{1}{n_t} \sum_{l=1}^j \sum_{k=1}^m S_{l,k} \quad (5)$$

$C_j$  is the probability that the tracer enters the spout at a  $z$  coordinate smaller than  $j \Delta$ . In other words,  $\mathbf{P}$  and  $\mathbf{C}$  give information about the spatial distribution of the tracer entrance in the spout.  $\mathbf{P}$  is called the vertical distribution of the mean radius of penetration of the solid particle in the spout, whereas  $\mathbf{C}$  is called the vertical distribution of the cumulative probability of the solid particles penetration in the spout.

A similar method is used to characterize the radial distribution of the mean height of penetration of the solid particles in the annulus and the radial distribution of the cumulative probability of the penetration of the solid particles in the annulus.

### Time distributions

For a given experiment during which the tracer experiences  $n_t$  cycles, the mean values of the solid particles residence times in the spout ( $t$ ), in the annulus ( $t$ ), and in the fountain ( $t$ ), and the mean value of the solid particle cycling time in the bed ( $t$ ) are defined as follows

$$\langle t_{sp} \rangle = \frac{1}{n_t} \sum_{j=1}^{n_t} \tau_{fo,i} - \tau_{sp,i} \quad (6)$$

$$\langle t_{an} \rangle = \frac{1}{n_t - 1} \sum_{j=1}^{n_t - 1} \tau_{sp,i+1} - \tau_{an,i} \quad (7)$$

$$\langle t_{fo} \rangle = \frac{1}{n_t} \sum_{j=1}^{n_t} \tau_{an,i} - \tau_{fo,i} \quad (8)$$

$$\langle t_c \rangle = \langle t_{sp} \rangle + \langle t_{an} \rangle + \langle t_{fo} \rangle \quad (9)$$

### Solid velocity

For a given experiment, the mean solid particles velocity in the annulus ( $v_{an}$ ) is defined as  $d_{an}/\langle t_{an} \rangle$ .

### Volumetric flow

For a given experiment, the global volumetric flow rate of the solid particles between adjacent regions of the bed ( $Q_v$ ) is defined as follows

$$Q_v = \frac{V_s}{\langle t_c \rangle} \quad (10)$$

where  $V_s$  is the total volume of solid particles in the bed.

The height of the window within which the solid particles migrate to the spout from the annulus ( $H_{a-s}$ ) is defined as follows

$$H_{a-s} = \frac{Q_v}{v_{an}} \frac{1}{2\pi r_{sp}} \quad (11)$$

### Development of correlations

Some empirical correlations may be obtained from the experimental data by applying dimensional analysis. A conical spouted bed is entirely characterized by nine operating parameters, seven of which are fixed in this study: the cone angle ( $\alpha$ ), the injection diameter ( $D_i$ ), the gas dynamic viscosity ( $\mu$ ), the gas density ( $\rho$ ), the solid particle diameter ( $d$ ), the density

of the solid particle ( $\rho$ ), the gravitational acceleration ( $g$ ). Two parameters vary between experiments: the inlet gas velocity ( $U$ ) and the static bed height ( $H_0$ ).

Any other characteristic parameters of the spouted bed may be expressed as a function of these nine operating parameters. In this study, four correlations are constructed, as functions of these nine operating parameters, for predicting the mean cycling time of the solid particles ( $\langle t_c \rangle$ ), the mean residence time of the solid particles in the annulus ( $\langle t_{an} \rangle$ ), the mean residence time of the solid particles in the spout ( $\langle t_{sp} \rangle$ ), and the global volumetric flow rate of the solid particles between adjacent regions of the bed ( $Q_v$ ).

Following the de Vaschy–Buckingham theorem, a relation between these nine operating parameters ( $\alpha$ ,  $D_i$ ,  $\mu$ ,  $\rho$ ,  $d$ ,  $\rho$ ,  $g$ ,  $U$ , and  $H_0$ ) and one of the other characteristic parameters of the spouted bed ( $\langle t_c \rangle$ ,  $\langle t_{an} \rangle$ ,  $\langle t_{sp} \rangle$ , or  $Q_v$ ) may be transformed in a relation between seven (nine minus two) dimensionless numbers (six involving exclusively the operating parameters and one involving the other characteristic parameter).

The six dimensionless numbers involving exclusively the operating parameters are chosen as follows:

- the angle  $\alpha$  (constant for the present experiments);
- the ratio  $d/D_i$  (constant for the present experiments);
- the ratio  $\rho/\rho_s$  (constant for the present experiments);
- the Archimedes number (with  $\rho \ll \rho_s$ ):  $Ar = \frac{gd_p^3 \rho \rho_s}{\mu^2}$  (constant for the present experiments);
- the ratio:  $H/D_i$ ;
- the relative gas injection velocity:  $U/U_{ms}$ .

$U_{ms}$  may be expressed as a function of  $\alpha$ ,  $D_i$ ,  $\mu$ ,  $\rho$ ,  $d$ ,  $\rho$ ,  $g$ , and  $H_0$  (through  $D_b$ ; see Eq. 1).  $U/U_{ms}$  is used as a dimensionless number because the results presented hereafter reveal that  $U_{ms}$  plays an important normalizing role when experiments with different values of  $H_0$  are compared.

Solely, the last two dimensionless numbers ( $H/D_i$  and  $U/U_{ms}$ ) may appear explicitly in the correlations developed in this work. Indeed, the other dimensionless numbers depend on geometrical or operating parameters that are held constant for all present experiments (see Table 1).

To develop these correlations, the following dimensionless numbers are defined

$$T_{sp} = \frac{\langle t_{sp} \rangle}{\left(\frac{D_i}{g}\right)^{0.5}} \quad (12)$$

$$T_{an} = \frac{\langle t_{an} \rangle}{\left(\frac{D_i}{g}\right)^{0.5}} \quad (13)$$

$$T_c = \frac{\langle t_c \rangle}{\left(\frac{D_i}{g}\right)^{0.5}} \quad (14)$$

$$Q = \frac{Q_v^2}{gD_i^5} \quad (15)$$

The following correlations are also introduced, the use of which will become clear in the results section

$$T_{sp} = a_1 \left(\frac{U}{U_{ms}}\right)^{a_2} \left(\frac{H_0}{D_i}\right)^{a_3} \quad (16)$$

$$T_{an} = b_1 \left(\frac{U}{U_{ms}}\right)^{b_2} \left(\frac{H_0}{D_i}\right)^{b_3} \quad (17)$$

$$T_c = c_1 \left(\frac{U}{U_{ms}}\right)^{c_2} \left(\frac{H_0}{D_i}\right)^{c_3} \quad (18)$$

$$Q = d_1 \left(\frac{U}{U_{ms}}\right)^{d_2} \left(\frac{H_0}{D_i}\right)^{d_3} \quad (19)$$

where  $a_i$ ,  $b_i$ ,  $c_i$ , and  $d_i$  are unknown parameters, either proportionality coefficients or exponents, that will be evaluated by means of least-square regression with respect to the experimental results.

## Results and Discussion

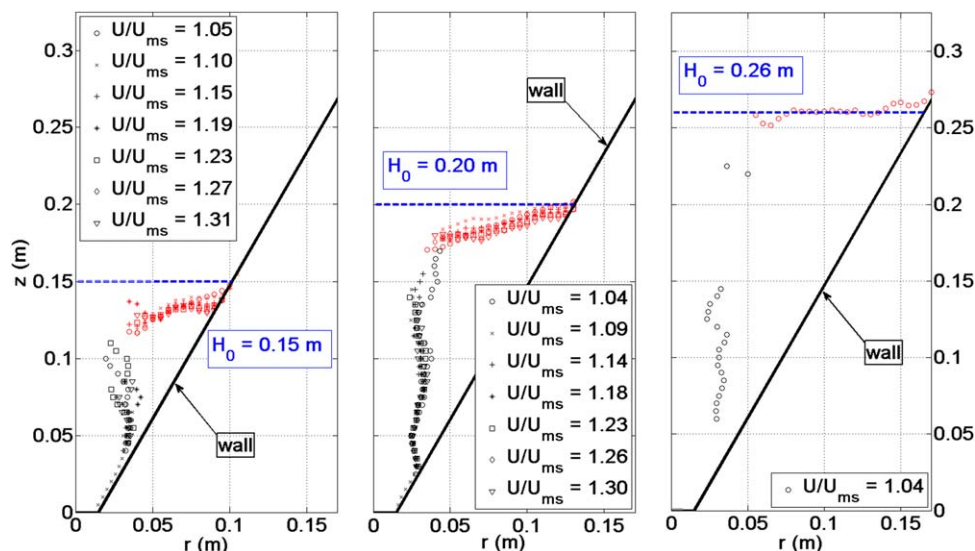
The method proposed in this article to characterize the solid particles flow in a conical spouted bed by tracking a single solid radioactive tracer, was applied to a set of experimental data. It allowed obtaining valuable results to quantify the influence of two operating parameters ( $U/U_{ms}$  and  $H_0$ ) on the solid particles flow behavior.

The vertical distribution of the mean radius of penetration of the solid particles in the spout ( $P$ ) and the radial distribution of the mean height of penetration of the solid particles in the annulus are presented in Figure 4, for different operating conditions. The spout (black markers) and the annulus (red markers) appear clearly on the figure; it is possible to visualize the shape of these two zones for the different operating conditions. It appears clearly that the shape of these two zones is not significantly influenced by  $U/U_{ms}$ , within the ranges of investigated operating conditions. Moreover, the spout radius ( $r_{sp}$ ) seems almost independent of the operating conditions, in the range of conditions investigated in this work (see Figure 4). In Figure 4, the spout cannot be visualized up to the annulus surface for all the operating conditions investigated, particularly when  $H_0$  is high, because there is no solid particles penetration from the annulus to the spout in the upper part of the spout.

The vertical distribution of the cumulative probability of the solid particles penetration in the spout ( $C$ ) is presented in Figure 5 for different operating conditions. This distribution appears to be only slightly influenced by  $U/U_{ms}$  under these operating conditions. For a given experiment, the mean value and the standard deviation of the height of the solid particles penetration in the spout ( $h_{sp}$  and  $\sigma$ ) are determined from the distribution  $C$ . For given values of  $H_0$ , the corresponding mean values of  $h_{sp}$  and  $\sigma$  are presented in Figure 5 [horizontal black line ( $h_{sp}$ ) and green dotted line ( $\sigma$ )] and in Table 2. It appears that for a fixed value of  $H_0$ ,  $\sigma$  is indeed almost independent of  $U/U_{ms}$  (see the low values of the relative standard deviations of  $\sigma$  presented in Table 2).

From Figure 5, it can also be observed that the solid particle penetration in the spout from the annulus occurs mostly on a limited height of the annulus-spout interface and that this penetration is relatively homogeneously distributed along this height. Consequently, the distribution of the solid particle penetration in the spout might be adequately modeled as being linear with respect to the vertical coordinate between the two boundary values of the height of the penetration window. These boundary values might be defined as  $h - \sigma$  and  $h + \sigma$  (see the red lines in Figure 5).

The height of the window of the solid particles penetration in the spout from the annulus ( $H_{a-s}$ ) was also calculated using Eq. 11 and is presented in Figure 5 (blue dashed lines). The values of  $H_{a-s}$  are also presented in Table 2 for the different values of  $H_0$ . It appears that, at a fixed value of  $H_0$ , this height



**Figure 4.** Vertical distribution of the mean radius of penetration of the solid particles in the spout (black markers) and radial distribution of the mean height of penetration of the solid particles in the annulus (red markers), for different operating conditions.

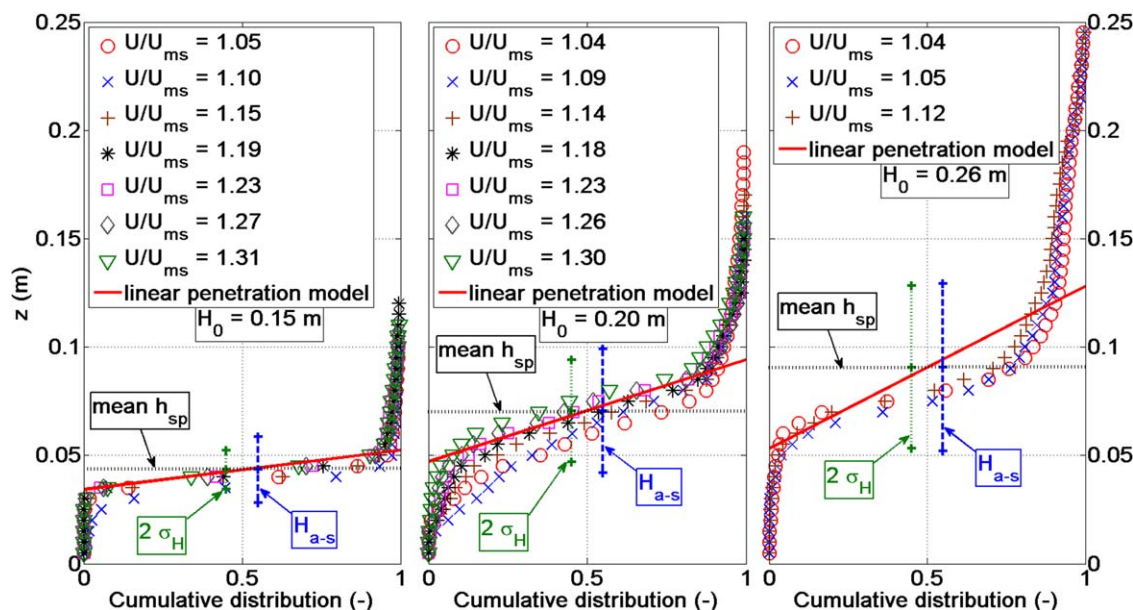
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is almost independent of  $U/U_{ms}$  (see the low values of the relative standard deviations of  $H_{a-s}$  presented in Table 2: between 4 and 7%). When  $H_0$  increases, the height of this window increases. It is worth mentioning that these evaluated values of  $H_{a-s}$  are very close to  $2\sigma$ . The difference between  $H_{a-s}$  and  $2\sigma$  is larger for low values of  $H_0$ , while it decreases when  $H_0$  increases. This similarity between  $H_{a-s}$  and  $2\sigma$  was of course physically expected and it highlights the adequacy of the data treatment method presented in this work, as similar results are obtained by two different analyses.

Similarly, the radial distribution of the cumulative probability of penetration of the solid particles in the annulus is presented in Figure 6, for different operating conditions. It

is observed that this distribution widens when  $H_0$  increases. This may be related to the fact that the area of the bed surface increases with higher  $H_0$  due to the conical shape of the bed, as illustrated by the increase of the stagnant conical bed diameter ( $D_b$ ) presented in Figure 6 (black vertical lines). For a given value of  $H_0$ , this distribution is almost independent of  $U/U_{ms}$ . Moreover, the penetration of the solid particles in the annulus appears to occur on the whole bed surface.

The mean values of  $r_{sp}$  for the given values of  $H_0$  are also presented in Figure 6 (red vertical dashed lines). These values are similar to the values predicted by the correlation published by San Jose et al.<sup>18</sup>



**Figure 5.** Vertical distribution of the cumulative probability of penetration of the solid particles in the spout, for different operating conditions.

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**Table 2. Twice the Mean Standard Deviation of the Vertical Distribution of the Probability of the Solid Particle Penetration in the Spout,  $2\sigma$ , and the Mean Height of the Window of the Solid Particle Penetration from the Annulus to the Spout,  $H_{a-s}$ , with Respect to the Stagnant Bed Height,  $H_0$**

	$H_0 = 0.15$ m	$H_0 = 0.20$ m	$H_0 = 0.26$ m
$2\sigma$ (cm)	$1.8 \pm 0.2$ (9%)	$4.7 \pm 0.5$ (11%)	$7.5 \pm 0.6$ (8%)
$H_{a-s}$ (cm)	$3.0 \pm 0.1$ (4%)	$5.7 \pm 0.1$ (2%)	$7.7 \pm 0.5$ (7%)

Analysis of the experimental results clearly highlights that  $U/U_{ms}$  does not influence significantly the solid bed morphology ( $r_{sp}$ ,  $h_{an}$ , spatial distributions of the transitions between zones *P* and *C*, and  $H_{a-s}$ ; see Figures 4–6 and Table 2). On the contrary,  $H_0$  has a large influence on this morphology. This indicates that the morphology of the solid bed is not strongly influenced by the forces exerted by the gas on the solid particles, but rather by geometrical considerations ( $D_i$ ,  $\alpha$ , amount of particles in the bed, etc.).

The mean solid particles velocity in the annulus ( $v_{an}$ ) is presented in Figure 7, for different operating conditions. It may be observed that  $v_{an}$  increases with increasing  $U/U_{ms}$  and that  $v_{an}$  does not seem to be influenced by  $H_0$ . This observation suggests that  $v_{an}$  is mostly controlled by interactions between solid particles inside the annulus.

The ratios of the mean residence time of the solid particles in the annulus ( $\langle t_{an} \rangle$ ), in the fountain ( $\langle t_{fo} \rangle$ ), and in the spout ( $\langle t_{sp} \rangle$ ) to the mean cycling time ( $\langle t_c \rangle$ ) are presented in Figure 8, for different operating conditions. It may be observed that the ratio of  $\langle t_{sp} \rangle$  to  $\langle t_c \rangle$  is almost independent of the operating conditions and is equal to approximately 0.077. This means that the solid particles spend approximately 8% of their time in the spout in all experiments. This ratio seems to be an intrinsic characteristic of the conical spouted bed, as it does not depend on the amount of particles in the bed and on the forces exerted by the gas on these particles.

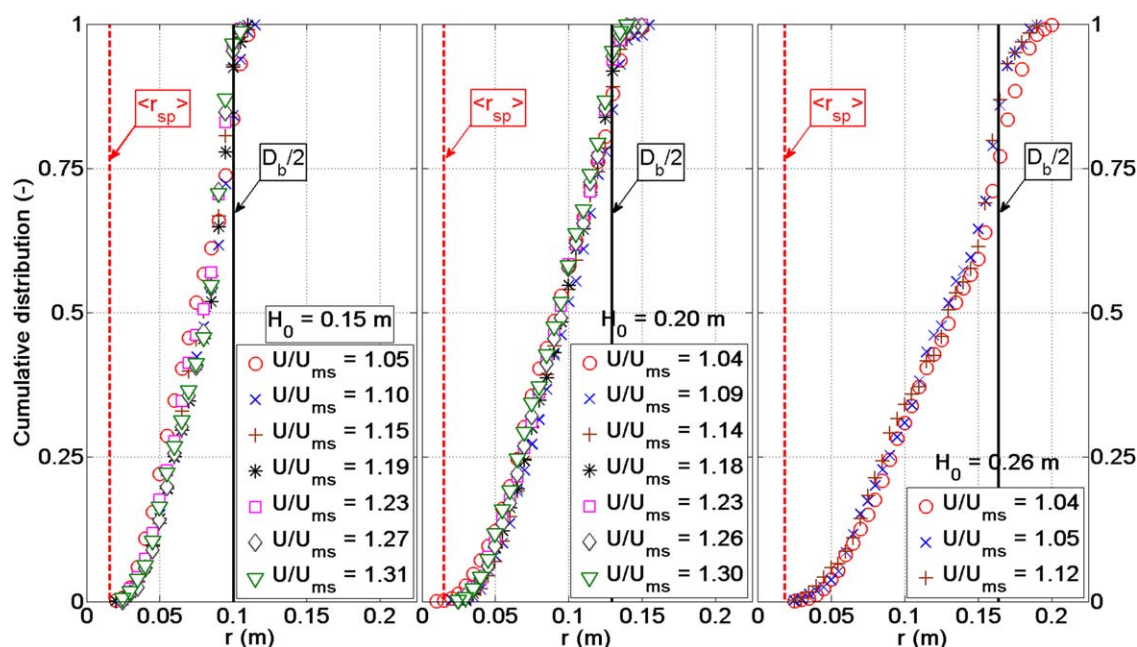
This value may change for other spouting regimes and conical bed geometry.

At a fixed value of  $H_0$ , the ratio of  $\langle t_{fo} \rangle$  to  $\langle t_c \rangle$ , that is, the fraction of the cycling time spent by the particles in the fountain, and therefore, the amount of particles in the fountain, increases when  $U/U_{ms}$  increases, which means that the fountain becomes more and more developed. In the fountain, due to the sudden decrease of the gas velocity, it may be assumed that gravity is the dominant force acting on the particles, and that, when they enter the fountain, the particles have a vertical velocity proportional to  $U$ . Therefore, these solid particles experience a uniform deceleration in the fountain, leading to a residence (flight) time proportional to their initial vertical velocity, that is, proportional to  $U$ . At the highest investigated values of  $U/U_{ms}$ , the fraction of the time spent in the fountain reaches approximately 40% for the shallowest bed (see Figure 8).

As a consequence of this and the fact that the ratio of  $\langle t_{sp} \rangle$  to  $\langle t_c \rangle$  is almost constant, the ratio of  $\langle t_{an} \rangle$  to  $\langle t_c \rangle$ , that is, the fraction of the cycling time spent by the solid particles in the annulus and, therefore, the amount of particles in the annulus, decreases when  $U/U_{ms}$  increases (see Figure 8), at a fixed value of  $H_0$ .

At a fixed value of  $U/U_{ms}$ , when  $H_0$  increases, the fraction of the cycling time spent in the annulus increases, while the fraction of the time spent in the fountain decreases. This may be explained by the fact that the mean residence times in the spout and in the fountain vary significantly less than the mean residence time in the annulus when  $H_0$  increases for a given value of  $U/U_{ms}$  (see Figures 9–11).

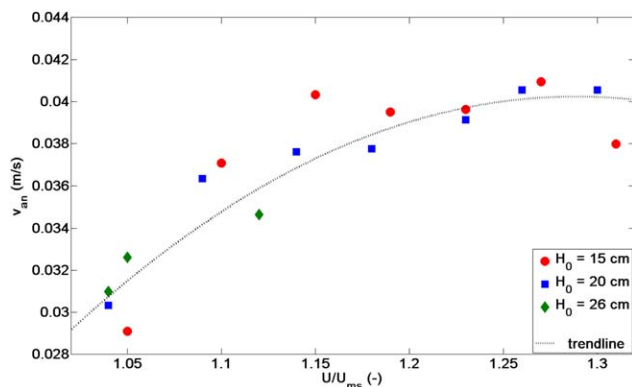
$Q_v \langle t_c \rangle$  is the total volume of solid particles in the bed (see Eq. 10), while  $Q_v \langle t_{an} \rangle$  is the volume of solid particles in the annulus. Therefore,  $\langle t_{an} \rangle / \langle t_c \rangle$  is the ratio of the volume of solid particles in the annulus to the total solid volume in the bed. Similar conclusions may be drawn for  $\langle t_{fo} \rangle / \langle t_c \rangle$  and  $\langle t_{sp} \rangle / \langle t_c \rangle$ . Then, as  $U/U_{ms}$  increases, the volume of solid particles in the annulus decreases, the volume of solid particles



**Figure 6. Radial distribution of the cumulative probability of penetration of the solid particles in the annulus, for different operating conditions.**

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**Figure 7.** Mean solid particle velocity in the annulus, as a function of  $U/U_{ms}$ , for different operating conditions.

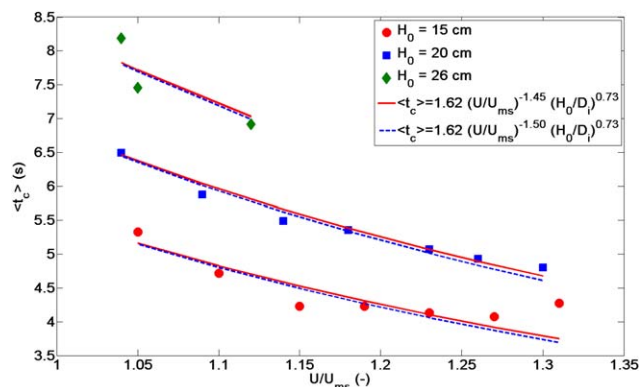
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in the fountain increases and the volume of solid particles in the spout remains constant (see Figure 8).

This decrease of the volume of solid particles in the annulus is a consequence of the development of the fountain. As the dimensions of the annulus are not significantly influenced by  $U/U_{ms}$ , as observed in Figure 4, an increase of  $U/U_{ms}$  then leads to an increase of the porosity of the annulus. This leads to a decrease in the interactions between particles in the annulus, and therefore, to an increase of  $v_{an}$  (see Figure 7). Moreover,  $H_0$  has an influence on the dimensions of the annulus (see Figure 4) but, as it is well known that the porosity is relatively homogeneous in the annulus region,<sup>19</sup>  $H_0$  has therefore no significant influence on the interactions between the particles therein, and then no significant influence on  $v_{an}$  (see Figure 7).

The unknown parameters appearing in the correlations (see Eqs. 16–19) proposed to express  $\langle t_c \rangle$ ,  $\langle t_{sp} \rangle$ ,  $\langle t_{an} \rangle$ , and  $Q_v$  as functions of  $U/U_{ms}$  and  $H_0$ , were evaluated through least-square regression with respect to the experimental results

$$\langle t_c \rangle = 1.62 \left( \frac{U}{U_{ms}} \right)^{-1.45} \left( \frac{H_0}{D_i} \right)^{0.73} \quad (20)$$



**Figure 9.** Mean cycling time of the solid particles in the conical spouted bed ( $\langle t_c \rangle$ ): comparison between the correlations and the experimental results, for different operating conditions ( $R^2 = 0.97$  for both correlations).

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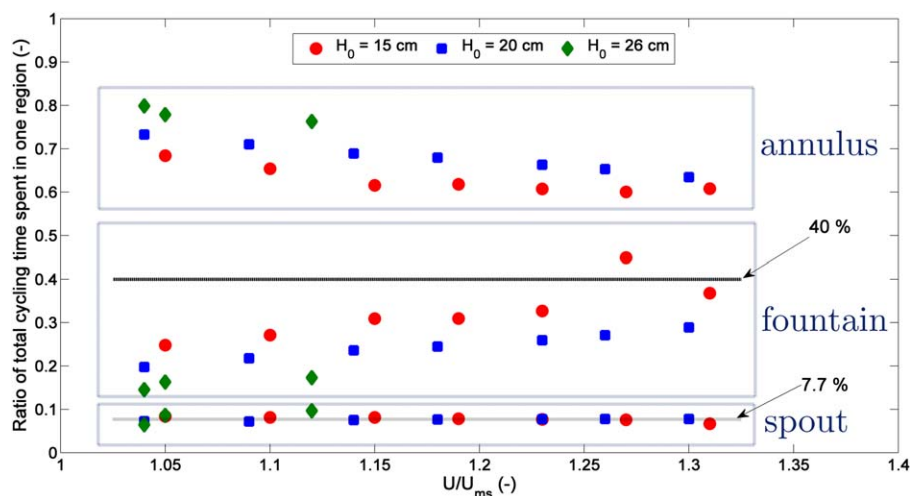
$$\langle t_{sp} \rangle = 0.075 \left( \frac{U}{U_{ms}} \right)^{-0.8} \left( \frac{H_0}{D_i} \right)^{0.96} \quad (21)$$

$$\langle t_{an} \rangle = 0.7 \left( \frac{U}{U_{ms}} \right)^{-2.15} \left( \frac{H_0}{D_i} \right)^{1.0} \quad (22)$$

$$Q_v = 6.42 \times 10^{-6} \left( \frac{U}{U_{ms}} \right)^{1.4} \left( \frac{H_0}{D_i} \right)^{2.0} \quad (23)$$

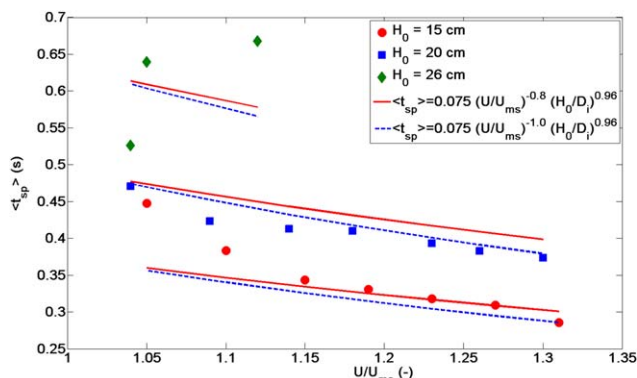
These correlations are presented in a dimensional form; the units are seconds ( $\langle t_c \rangle$ ,  $\langle t_{sp} \rangle$ , and  $\langle t_{an} \rangle$ ) and cubic meter per second ( $Q_v$ ). The ranges of values of  $U/U_{ms}$  and  $H_0/D_i$  where the correlations are valid are [1.0, 1.3] and [5.35, 9.3], respectively. These upper limit values were imposed by the air compressor limitations.

Figures 9–12 (red solid lines) show how these correlations and the experimental results are in very good agreement, meaning that these correlations allow a good fitting of the characteristic times of the solid flow in the conical spouted bed and the solid particles flow rate between adjacent regions of the bed. In these figures, the dashed blue lines present the



**Figure 8.** Ratios of the total cycling time spent by the solid particles in the annulus, the spout and the annulus, for the different operating conditions investigated.

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**Figure 10.** Mean residence time of the solid particles in the spout of the conical spouted bed ( $\langle t_{sp} \rangle$ ): comparison between the correlations and the experimental results, for different operating conditions ( $R^2 = 0.81$  for both correlations).

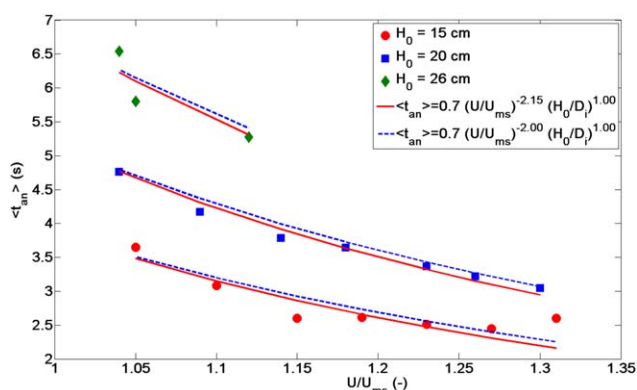
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correlations when the evaluated parameters are rounded to the closest integer or half-integer values and show that the agreement remains very good in all cases.

In the ranges of investigated values of  $U/U_{ms}$  and  $H_0/D_i$ , which are [1.0,1.3] and [5.35,9.3], respectively, the ranges of values measured for  $\langle t_c \rangle$ ,  $\langle t_{sp} \rangle$ , and  $\langle t_{an} \rangle$  are [4,8], [0.3,0.65], and [2.5,6.5], respectively.

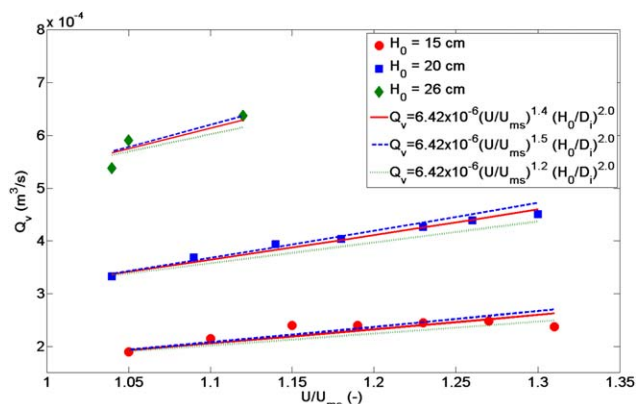
In the spout, it may be supposed that the drag force acting on the particles (proportional to  $U^2$ ) is the dominant force, due to the large value of  $U$ . Therefore, the solid particles have a uniform acceleration in the spout, leading, at a fixed value of  $H_0$  (and, therefore, at a fixed distance traveled in the spout), to values of  $\langle t_{sp} \rangle$  inversely proportional to the square root of the force acting on these particles, that is, inversely proportional to  $U$ . This may explain that the exponent of  $U/U_{ms}$  in Eq. 21 is close to  $-1$ .

At a fixed value of  $U/U_{ms}$ , as  $v_{an}$  is independent of  $H_0$  (see Figure 7) and as the average distance traveled by the solid par-



**Figure 11.** Mean residence time of the solid particles in the annulus of the conical spouted bed ( $\langle t_{an} \rangle$ ): comparison between the correlation and the experimental results, for different operating conditions ( $R^2 = 0.98$  for both correlations).

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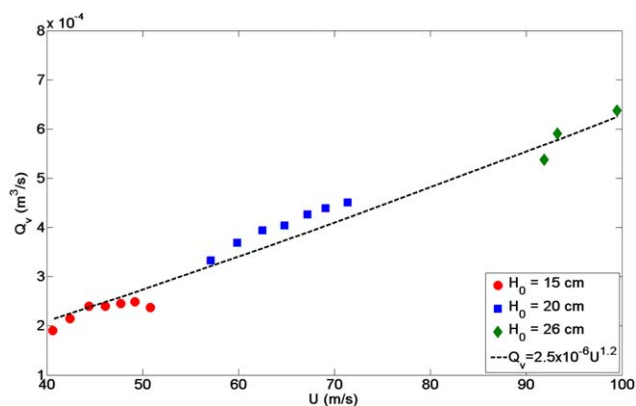
**Figure 12.** Volumetric flow rate of the solid particles between adjacent regions of the bed ( $Q_v$ ): comparison between the correlation and the experimental results, for different operating conditions ( $R^2 = 0.99$  for the three correlations).

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ticles in the annulus is proportional to mean annulus height  $h_{an}$ , the mean residence time in the annulus is proportional to  $h_{an}$ , which is itself proportional to  $H_0$  (see Figure 4). This may explain why the exponent of  $H_0$  in Eq. 22 is equal to 1.

At a fixed value of  $H_0$ , the value of  $Q_v$  increases when  $U/U_{ms}$  increases (see Figure 12). This increase is not due to an increase of the crossflow area between the annulus and the spout ( $2 \pi r_{sp} H_{a-s}$ ), as  $H_{a-s}$  and  $r_{sp}$  are almost independent of  $U/U_{ms}$  (see Table 2 and Figure 4). Therefore, this increase may only be explained by an increase of the solid particles superficial velocity across the crossflow area between the annulus and the spout, which is consistent with the results obtained for  $v_{an}$  (see Figure 7).

At a fixed value of  $U/U_{ms}$ , the value of  $Q_v$  increases when  $H_0$  increases (see Figure 12). As  $v_{an}$  is almost constant if  $U/U_{ms}$  is fixed (see Figure 7), this increase is only due to the increase of the crossflow area between the annulus and the spout when  $H_0$  increases (see Table 2).



**Figure 13.** Volumetric flow rate of the solid particles between adjacent regions of the bed, presented as a function of the inlet gas velocity, for different operating conditions.

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In Figure 12, two alternative correlations are presented with two different values of the exponent of  $U/U_{ms}$ : 1.5 and 1.2, instead of 1.4. From this sensitivity analysis, it appears that both alternative correlations do not significantly affect the agreement with the experimental results which means that the correlation validity is not significantly influenced by the value of the exponent of  $U/U_{ms}$  in the range [1.2,1.5].

If  $Q_v$  is presented as a function of  $U$ , an almost linear relation between these two parameters, nearly independent of  $H_0$ , may be observed (see Figure 13). It means that it is the force exerted on the solid particles by the gas that directly controls  $Q_v$ , and not the amount of particles in the bed. It is worth mentioning that the independence of  $Q_v$  with respect to  $H_0$  may be further appreciated by introducing the fact that  $U_{ms}$  is proportional to  $H_0^{1.68}$  (see Eq. 1) in the correlation for  $Q_v$  expressed with an exponent for  $U/U_{ms}$  equal to 1.2 (see Figure 12)

$$Q_v \propto \left(\frac{U}{U_{ms}}\right)^{1.2} (H_0)^{2.0} \propto \left(\frac{U}{H_0^{1.68}}\right)^{1.2} (H_0)^{2.0} \approx U^{1.2} (H_0)^{0.0} \quad (24)$$

## Conclusion

In this work, a method is proposed to characterize the solid particles flow in a conical spouted bed by tracking a single solid radioactive tracer. This method is composed of two steps. In a first step, from the raw experimental data, the different transition times of the solid particles between adjacent regions of the bed are evaluated, based on a definition of these transition times. In a second step, several key parameters of the solid particles flow in the conical spouted bed are calculated, from the knowledge of these transition times and the corresponding particle positions, for the different cycles of an experiment. These key parameters include the bed dimensions, spatial distributions, mean residence times in the different regions of the bed, and volumetric flow rate of the solid particles between adjacent regions of the bed. This method was applied to experimental data collected for various operating conditions of a conical spouted bed. The results allowed highlighting key phenomena governing this solid particle flow in the conical spouted bed.

The solid particles crossflow between the annulus and the spout occurs on a limited section of the interface between these regions. The height of the window of penetration in the spout from the annulus is independent of  $U/U_{ms}$  for a given value  $H_0$  and increases when  $H_0$  increases. The spatial distribution of the solid particles penetration through this window is found to be homogeneous. A similar independence of the spatial distribution of penetration in the annulus with respect to  $U/U_{ms}$  for a given  $H_0$  is also shown.

The morphology of the solid bed is not strongly influenced by the forces exerted by the gas on the solid particles (independence with respect to  $U/U_{ms}$ ), but rather by geometrical considerations (strong dependence on  $H_0$ ).

The mean solid particle velocity in the annulus is mostly controlled by the interactions between the solid particles inside the annulus (dependence on  $U/U_{ms}$  but no significant dependence on  $H_0$ ).

The solid particles spend approximately 8% of their time in the spout in all experiments ( $\langle t_{sp} \rangle / \langle t_c \rangle = 0.077$ ). This ratio seems to be an intrinsic characteristic of the conical spouted bed used in this study, as it does not depend on the amount of particles in the bed or on the forces exerted by the gas of these particles. This result is very useful in the modeling of spouted

bed applications and has been successfully used in a recent publication about drying Baker's yeast in a conical spouted bed.<sup>20</sup>

The force exerted on the solid particles by the gas directly controls  $Q_v$ , and not the amount of particles in the bed.

The mean residence times in the spout and in the fountain vary significantly less than the mean residence time in the annulus when  $H_0$  increases for a given value of  $U/U_{ms}$ .

The amount of solid particles in the fountain increases when  $U/U_{ms}$  increases, which means that the fountain becomes more and more developed. At the highest investigated values of  $U/U_{ms}$ , the fraction of the time spent in the fountain reaches approximately 40%. On the contrary, the amount of solid particles in the annulus decreases when  $U/U_{ms}$  increases, at a fixed value of  $H_0$ .

As  $U/U_{ms}$  increases, the volume of solid particles in the annulus decreases, the volume of solid particles in the fountain increases and the volume of solid particles in the spout remains constant.

Correlations to predict key parameters of this solid particle flow as a function of the operating conditions were also established and discussed.

The results presented in this work contribute to a better understanding of the phenomena governing the solid particles flow in a conical spouted bed. They enable the development of models describing this flow that could be coupled with a model of the gas flow in a conical spouted bed published elsewhere.<sup>21</sup> These results may also lead to the development or the improvement of numerical simulations of the gas-solid flow in conical spouted beds.

## Acknowledgments

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## Notation

### Symbols

- $a_i, b_i, c_i, d_i$  = unknown parameters, either proportionality coefficients or exponents in correlations
- Ar = Archimedes number
- $C$  = vector of the vertical distribution of the cumulative probability of the penetration of the solid particles in the spout
- $d_{an}$  = mean distance traveled by the solid particle in the annulus, m
- $D_b$  = diameter of the stagnant bed surface, m
- $D_c$  = diameter of the cylinder, m
- $D_i$  = diameter of the gas inlet, m
- $D_{max}$  = maximum diameter of the cone, m
- $d_p$  = diameter of the solid particle, m
- $d_{sp}$  = mean distance traveled by the solid particle in the spout, m
- $g$  = gravitational acceleration, m/s<sup>2</sup>
- $H_0$  = static bed height, m
- $h_{an}$  = mean height of the annulus, m
- $H_{a-s}$  = height of the window of the solid particles penetration in the spout from the annulus, m
- $H_b$  = height of the cone, m
- $H_c$  = height of the cylinder, m
- $h_{sp}$  = mean height of the solid particles penetration in the spout, m
- $(j, k)$  = coordinates of the elements of the discretized domain
- $n \times m$  = number of elements of the discretized domain
- $n_t$  = number of cycles in an experiment
- $P$  = vector of the vertical distribution of the mean radius of penetration of the solid particles in the spout

$Q$  = dimensionless number  
 $Q_v$  = volumetric flow rate of the solid particles between adjacent regions of the bed,  $\text{m}^3/\text{s}$   
 $r$  = radial coordinate, m  
 $\mathbf{R}$  = vector of the radial coordinate of the center of the elements of the discretized domain  
 $Re_{\text{msi}}$  = Reynolds number for minimum spouting velocity based on  $D_i$   
 $r_{\text{sp}}$  = mean radius of the spout, m  
 $\mathbf{S}$  = matrix of the transition occurrences in each element of the discretized domain  
 $T_{\text{an}}, T_c, T_{\text{sp}}$  = dimensionless numbers  
 $\langle t_{\text{an}} \rangle, \langle t_{\text{fo}} \rangle, \langle t_{\text{sp}} \rangle$  = mean values of the solid particles residence times in the annulus, fountain, and spout, s  
 $\langle t_c \rangle$  = mean value of the solid particle cycling time in the bed, s  
 $U$  = superficial gas velocity based on  $D_i$  and expressed in normal conditions ( $20^\circ\text{C}$  and 1 atm),  $\text{Nm/s}$   
 $U_{\text{ms}}$  = superficial gas velocity at minimum spouting based on  $D_i$  and expressed in normal conditions ( $20^\circ\text{C}$  and 1 atm),  $\text{Nm/s}$   
 $v_{\text{an}}$  = mean velocity of the solid particles in the annulus,  $\text{m/s}$   
 $V_s$  = total volume of solid particles in the bed,  $\text{m}^3$   
 $z$  = vertical coordinate, m  
 $\alpha$  = cone angle  
 $\Delta$  = size of the side of square discretization element, mm  
 $\mu$  = gas dynamic viscosity, Pa s  
 $\rho$  = gas density,  $\text{kg/m}^3$   
 $\rho$  = solid particle density,  $\text{kg/m}^3$   
 $\sigma$  = standard deviation of  $h_{\text{sp}}$ , m  
 $\tau_{\text{an},i}, \tau_{\text{fo},i}, \tau_{\text{sp},i}$  = transition time from the fountain to the annulus, from the spout to the fountain and from the annulus to the spout during the  $i$ th cycle of an experiment, s

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